Eötvös Mathematical Competition 1905

1. Find the necessary and sufficient conditions on positive integers n, p for the system of equations

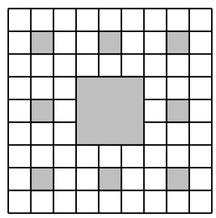
$$x + py = n, \quad x + y = p^z$$

to have a positive integral solution (x, y, z). Also prove that there is at most one such solution.

2. Divide the unit square into 9 equal squares and remove the central square. Now treat each of the remaining 8 squares the same way, and repeat this process n times.

   (a) How many squares of side length $1/3^n$ remain?

   (b) What is the limit sum of the areas of the removed squares as n approaches infinity?



3. Let C_1 be any point on side AB of a triangle ABC. The lines through A and B parallel to CC_1 intersect the lines BC and AC respectively at A_1 and B_1. Prove that

$$\frac{1}{AA_1} + \frac{1}{BB_1} = \frac{1}{CC_1}$$

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