Eötvös Mathematical Competition 1903

1. Suppose that $2^p - 1$ is a prime number. Prove that the sum of all positive divisors of $n = 2^{p-1}(2^p - 1)$ (excluding n) is exactly n.

2. For a given pair of values x and y satisfying $x = \sin\alpha$, $y = \sin\beta$, there can be four different values of $z = \sin(\alpha + \beta)$.

   (a) Set up a relation between x, y and z not involving trigonometric functions or radicals.

   (b) Find those pairs of values (x, y) for which $z = \sin(\alpha + \beta)$ assumes fewer than four distinct values.

3. For a rhombus $ABCD$, let k_1 be the circle through B, C, D, k_2 be the circle through A, C, D, k_3 be the circle through A, B, D, and k_4 be the circle through A, B, C. Prove that the tangents to k_1 and k_3 at B form the same angle as the tangents to k_2 and k_4 at A.

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