Eötvös Mathematical Competition 1902

1. Consider an arbitrary quadratic polynomial $Q(x) = Ax^2 + Bx + C$.

   (a) Prove that $Q(x)$ can be written in the form

   $$Q(x) = k\frac{x(x-1)}{1 \cdot 2} + lx + m$$

   where k, l, m depend on the coefficients A, B, C.

   (b) Prove that $Q(x)$ takes integral values for every integer x if and only if k, l, m are integers.

2. Let S be a given sphere with center O and radius r, and P be a point outside S. Sphere S' has center P and radius PO. Denote by $\mathcal{F}$ the area of the surface of the part of S' that lies inside S. Prove that $\mathcal{F}$ is independent of point P.

3. The area T and an angle γ of a triangle are given. Find the side lengths a and b so that the side c opposite γ is shortest possible.

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