

## 2-nd Hong Kong (China) Mathematical Olympiad 1999

December 18, 1999

1. Find all positive rational numbers  $r \neq 1$  such  $r^{\frac{1}{r-1}}$  is rational. [5 pts]
2. Let  $I$  be the incenter and  $O$  the circumcenter of a non-equilateral triangle  $ABC$ . Prove that  $\angle AIO \leq 90^\circ$  if and only if  $2BC \leq AB + AC$ . [10 pts]
3. Students have taken a test paper in each of  $n \geq 3$  subjects. It is known that in any subject exactly three students got the best score, and for any two subjects exactly one student got the best scores in both subjects. Find the smallest  $n$  for which the above conditions imply that exactly one student got the best score in each of the  $n$  subjects. [10 pts]
4. Determine all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x + yf(x)) = f(x) + xf(y) \quad \text{for all } x, y \in \mathbb{R}. \quad [10 \text{ pts}]$$