

# 6-th Hong Kong (China) Mathematical Olympiad 2003

December 20, 2003

1. Find the greatest real number  $K$  such that for all positive numbers  $u, v, w$  with  $u^2 > 4vw$  we have

$$(u^2 - 4vw)^2 > K(2v^2 - uv)(2w^2 - uv).$$

2. Let  $ABCD$  be a regular hexagon of side length 1, and  $O$  be its center. In addition to the sides of the hexagon, line segments from  $O$  to each vertex are drawn, making a total of 12 unit segments. Find the number of paths of length 2003 along these segments that start at  $O$  and terminate at  $O$ .
3. Let  $K, L, M, N$  be the midpoints of sides  $AB, BC, CD, DA$  of a cyclic quadrilateral  $ABCD$ . Prove that the orthocenters of triangles  $ANK, BKL, CLM, DMN$  are the vertices of a parallelogram.
4. Determine all integers  $a, b, c$  such that

$$\frac{1}{2}(a+b)(b+c)(c+a) + (a+b+c)^3 = 1 - abc.$$