4-th Hong Kong (China) Mathematical Olympiad 2001

1. A triangle ABC is given. A circle Γ, passing through A, is tangent to side BC at point P and intersects sides AB and AC at M and N respectively. Prove that the smaller arcs MP and NP of Γ are equal if and only if Γ is tangent to the circumcircle of $\triangle ABC$ at A.

2. Find, with proof, all positive integers n such that the equation

$$x^3+y^3+z^3=nx^2y^2z^2$$

has a solution in positive integers.

3. Let $k \geq 4$ be an integer. Prove that if $P(x)$ is a polynomial with integer coefficients such that $0 \leq F(c) \leq k$ for $c=0,1,\dots,k+1$, then $F(0)=F(1)=\cdots=F(k+1)$.

4. There are 212 points inside or on a given unit circle. Prove that there are at least 2001 pairs of points having distances at most 1.