6-th Hungary–Israel Binational Mathematical Competition 1995

1. Let S_n be the sum of the first n prime numbers. Prove that there is a perfect square between S_n and S_{n+1}.

2. Let P, P_1, P_2, P_3, P_4 be five points on a circle, and let d_{ik} denote the distance of P from the line P_iP_k. Prove that $d_{12}d_{34} = d_{13}d_{24}$.

3. Consider the polynomials $f(x) = ax^2 + bx + c$ with real coefficients which satisfy $|f(x)| \leq 1$ for $0 \leq x \leq 1$. Find the maximum value of $|a| + |b| + |c|$.

4. All faces of a convex polyhedron are triangles. Prove that it is possible to color the edges by red and blue so that, for any of the two colors, one can travel from any vertex to any other vertex passing only through edges of that color.

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