2-nd Hungary–Israel Binational Mathematical Competition 1991

1. Suppose $f(x)$ is a polynomial with integer coefficients such that $f(0) = 11$ and $f(x_1) = f(x_2) = \cdots = f(x_n) = 2002$ for some distinct integers $x_1, x_2, \dots, x_n$. Find the largest possible value of n.

2. A rectangular sheet of paper is folded so that point D maps to a point D' on side BC. Thereby point A maps to a point A'. The lines AB and $A'D'$ intersect at E. Prove that if r is the inradius of the triangle EBD', then $r = A'E$.

3. Let H_n be the set of numbers $2 \pm \sqrt{2 \pm \sqrt{2 \pm \cdots \pm \sqrt{2}}}$ with n square roots.

   (a) Prove that all elements of H_n are real numbers.

   (b) Evaluate the product of the elements of H_n.

   (c) If the elements of H_{11} are arranged in the increasing order, find the position of the element determined by the sequence of signs $+ + + + + - + + - + -$.

4. Find all real values of λ for which the system

$$\begin{aligned} x + y + z + v &= 0 \\ (xy + yz + zv) + \lambda(xz + xv + yv) &= 0 \end{aligned}$$

has a unique real solution.

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