13-th Hungary–Israel Binational Mathematical Competition 2002

First Day – Budapest, March 21

1. Find the greatest exponent k for which 2001^k divides $2000^{2001^{2002}} + 2002^{2001^{2000}}$.

2. Points A_1, B_1, C_1 are given inside an equilateral triangle ABC such that

$$\angle B_1AB = \angle A_1BA = 15^\circ,$$
$$\angle C_1BC = \angle B_1CB = 20^\circ,$$
$$\angle A_1CA = \angle C_1AC = 25^\circ.$$

   Find the angles of triangle $A_1B_1C_1$.

3. Let $p \geq 5$ be a prime number. Prove that there exists a positive integer $a < p-1$ such that neither of $a^{p-1}-1$ and $(a+1)^{p-1}-1$ is divisible by p^2.

Second Day – Budapest, March 22

4. Suppose that positive numbers x and y satisfy $x^3+y^4 \leq x^2+y^3$. Prove that $x^3+y^3 \leq 2$.

5. Let A', B', C' be the projections of a point M inside a triangle ABC onto the sides BC, CA, AB, respectively. Define $p(M) = \frac{MA' \cdot MB' \cdot MC'}{MA \cdot MB \cdot MC}$. Find the position of point M that maximizes $p(M)$.

6. Let $p(x)$ be a polynomial with rational coefficients, of degree at least 2. Suppose that a sequence (r_n) of rational numbers satisfies $r_n = p(r_{n+1})$ for every $n \geq 1$. Prove that the sequence (r_n) is periodic.

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