Greek Team Selection Test 1998

1. If $x, y, z > 0$, $k > 2$ and $a = x + ky + kz$, $b = kx + y + kz$, $c = kx + ky + z$, show that
$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} \geq \frac{3}{2k+1}.$$

2. Let $ABCD$ be a trapezoid with $AB \parallel CD$ and M, N be points on the lines AD and BC respectiveyl such that $MN \parallel AB$. Prove that
$$DC \cdot MA + AB \cdot MD = MN \cdot AD.$$

3. Prove that if the number $A = 111\dots1$ (n digits) is prime, then n is prime. Is the converse true?

4. (a) A polynomial $P(x)$ with integer coefficents takes the value -2 for at least seven distinct integers x. Prove that it cannot take the value 1996.

   (b) Prove that there are irrational numbers x, y such that the number x^y is rational.

5. Let I be an open interval of length $1/n$, where $n \in \mathbb{N}$. Find the maximum possible number of rational numbers of the form a/b, where $1 \leq b \leq n$, that lie in I.

6. The sum of k different even and l different odd natural numbers equals 1997. Find the maximum possible value of $3k + 4l$.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com