14-th Hellenic Mathematical Olympiad 1997

Seniors

1. Let P be a point inside or on the boundary of a square $ABCD$. Find the minimum and maximum values of

$$f(P) = \angle ABP + \angle BCP + \angle CDP + \angle DAP.$$

2. Let a function $f : \mathbb{R}^+ \to \mathbb{R}$ satisfy:

   (i) f is strictly increasing;

   (ii) $f(x) > -1/x$ for all $x > 0$;

   (iii) $f(x)f\left(f(x)+1/x\right) = 1$ for all $x > 0$.

   Determine $f(1)$.

3. Find all integer solutions to $\dfrac{13}{x^2} + \dfrac{1996}{y^2} = \dfrac{z}{1997}$.

4. A polynomial P with integer coefficients has at least 13 distinct integer roots.Prove that if an integer n is not a root of P, then $|P(n)| \geq 7 \cdot 6!^2$, and give an example for equality.

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The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
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