13-th Hellenic Mathematical Olympiad 1996

Seniors

1. In a triangle ABC, points D, E, Z, H, Θ are the midpoints of segments BC, AD, BD, ED, EZ, respectively. Lines BE and $H\Theta$ intersect AC at I and K, respectively. Prove that:

   (a) $AK = 3CK$;

   (b) $HK = 3H\Theta$;

   (c) $BE = 3EI$;

   (d) the area of ABC is 32 times the area of $E\Theta H$.

2. In a triangle ABC, AD, BE, CZ are the altitudes and H the orthocenter. Let AI and $A\Theta$ be the internal and external bisectors of angle A, and let M, N be the midpoints of BC, AH, respectively.

   (a) Prove that MN is perpendicular to EZ.

   (b) Prove that if MN meets AI and $A\Theta$ at K and L, then $KL = AH$.

3. Prove that among 81 natural numbers whose prime divisors are in the set $\{2,3,5\}$ there exist four numbers whose product is the fourth power of an integer.

4. Find the number of functions $f : \{1,2,\dots,n\} \to \{1995,1996\}$ such that $f(1)+f(2)+\cdots+f(1996)$ is odd.

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