27-th German Federal Mathematical Competition 1996/97

Second Round

1. Three faces of a regular tetrahedron are painted in white and the remaining one in black. Initially, the tetrahedron is positioned on a plane with the black face down. It is then tilted several times over its edges. After a while it returns to its original position. Can it now have a white face down?

2. Show that for any rational number a the equation $y = \sqrt{x^2 + a}$ has infinitely many solutions in rational numbers x and y.

3. A semicircle with diameter $AB = 2r$ is divided into two sectors by an arbitrary radius. To each of the sectors a circle is inscribed. These two circles touch AB at S and T. Show that $ST \geq 2r(\sqrt{2} - 1)$.

4. Prove that if n is a natural number such that both $3n + 1$ and $4n + 1$ are squares, then n is divisible by 56.

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