# 11-th German Federal Mathematical Competition 1980/81

## Second Round

1. A sequence $a_1, a_2, a_3, \dots$ is defined as follows: $a_1$ is a positive integer and $a_{n+1} = [\frac{3}{2}a_n] + 1$ for all $n \in \mathbb{N}$. Can $a_1$ be taken in such a way that the first 100000 terms of the sequence are even, but the 100001-th term is odd?

2. A bijective mapping from a plane to itself maps every circle to a circle. Prove that it maps every line to a line.

3. Let $n = 2^{k-1}$, where $k$ is a positive integer. Prove that among any $2n-1$ integers there exist $n$ integers whose sum is divisible by $n$.

4. A set $M$ of natural numbers has the property that for every $x \in M$, $4x$ and $[\sqrt{x}]$ are elements of $M$. Show that every natural number belongs to $M$.



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