8-th German Federal Mathematical Competition 1977/78

Second Round

1. Let a,b,c be sides of a triangle. Denote $R = a^2 + b^2 + c^2$ and $S = (a+b+c)^2$. Prove that $\frac{1}{3} \leq \frac{R}{S} < \frac{1}{2}$, and show that $1/2$ cannot be replaced with a smaller number.

2. Seven distinct points are given inside a square with side 1. Together with the vertices of the square they form a set of 11 points. Consider all triangles with vertices in M.

   (a) Show that at least one of these triangles has an area not exceeding $1/16$.

   (b) Give an example in which no four of the seven points are on a line and none of the considered triangles has an area less than $1/16$.

3. S unn and Tacks play a game alternately choosing a word among the following (German) words: ”bad”, ”binse”, ”k afig”, ”kosewort”, ”maitag”, ”name”, ”pol”, ”parade”, ”wolf”. Two words are said to *compatible* if they have exactly one consonant in common. In the first round, S unn selects a word for herself and one for Tacks. In every consequent round, each player selects a word he/she had in the previous round, with S unn playing first. Tacks wins the game if the two players successively select the same word.

   (a) Prove that Tacks can always win. How many rounds are necessary for that?

   (b) Upon S unn’s desire, the word ”kafig” was replaced with the word ”feige”. Prove that Su nn can prevent Tacks from winning.

4. A prime number has the property that however its decimal digits are permuted, the obtained number is also prime. Prove that this number has at most three different digits. Also prove a stronger statement.

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