6-th German Federal Mathematical Competition 1975/76

Second Round

1. Prove that if n is an odd natural number, then $1^n + 2^n + \cdots + n^n$ is divisible by n^2.

2. Two congruent squares Q and Q' are given in the plane. Show that they can be divided into parts $T_1, T_2, \ldots, T_n$ and $T_1', T_2', \ldots, T_n'$, respectively, such that T_i' is the image of T_i under a translation V_i for $i = 1, \ldots, n$.

3. A circle is divided by n points into n equal arcs. Let $P_1, P_2, \ldots, P_n$ be an arbitrary permutation of the n division points. Prove that the polygonal line $P_1P_2 \ldots P_{2n}P_1$ contains at least two parallel segments.

4. Each point in space is colored either red or blue. Prove that there exists a square with side 1 having either three red vertices or four blue vertices.

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