

1-st German Federal Mathematical Competition 1970/71

Second Round

1. Let a, b, c, d be natural numbers with $ab = cd$. Prove that $a^2 + b^2 + c^2 + d^2$ cannot be a prime number. Formulate and prove a generalization of this statement.
2. The inhabitants of a certain planet speak a language whose words consist of letters A and O only. For the aim of avoidance of errors, every two words of the same length differ in at least three positions. Prove that there are no more than $\frac{2^n}{n+1}$ words of length n .
3. Every two towns in a country are connected by a one-way road. Prove that there is a town which can be reached from any other town either directly or passing through at most one other town.
4. A non-selfintersecting polygonal line of a length exceeding 1000 is given inside a unit square. Show that for every such polygonal line, there is a line parallel to one of the sides of the square that intersects the polygonal line in at least 501 points.