# 32-nd German Federal Mathematical Competition 2001/02

## Second Round

1. A deck of cards which are labelled with numbers from 1 to $n$ is shuffled. The following procedure is applied repeatedly: If the card with number $k$ is at the top of the deck, then the order of the top $k$ cards is reversed. Prove that after finitely many steps the card with number 1 will be at top position.

2. Consider strictly increasing sequences $a_0, a_1, a_2, \dots$ of nonnegative integers with the property that every nonnegative integer can be uniquely written in the form $a_i + 2a_j + 4a_k$ (with $i, j, k$ not necessarily distinct). Show that there exists exactly one such sequence and determine $a_{2002}$.

3. Given a convex polyhedron with an even number of edges, prove that every edge can be assigned an arrow such that for each vertex the number of incoming arrows is even.

4. In an acute-angled triangle $ABC$, $H_a$ and $H_b$ are the feet of the altitudes from $A$ and $B$, respectively. Furthermore, $W_a$ and $W_b$ are the intersections of the angle bisectors of $\angle CAB$ and $\angle ABC$ with the opposite sides, respectively. Prove that the incenter $I$ of $\triangle ABC$ lies on segment $H_aH_b$ if and only if the circumcenter $O$ lies on segment $W_aW_b$.



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