

# 31-st German Federal Mathematical Competition 2000/01

## Second Round

1. Ten of the vertices of a regular 100-gon are colored red and ten other vertices are colored blue. Prove that there exist a segment connecting two red vertices and a segment of the same length connecting two blue vertices.
2. For every natural number  $n \geq 0$ , find two integers  $p_n$  and  $q_n$  with the following property: There are exactly  $n$  integers  $x$  for which the number  $x^2 + p_n x + q_n$  is a perfect square.
3. Points  $A'$ ,  $B'$  and  $C'$  are taken on the sides  $BC$ ,  $CA$  and  $AB$ , respectively, such that

$$A'B' = B'C' = C'A' \quad \text{and} \quad AB' = BC' = CA'.$$

Prove that triangle  $ABC$  is equilateral.

4. Inside a square  $\mathcal{Q}$  with side length 500 lies a square  $\mathcal{R}$  with side length 250. Prove that there always exist two points  $A, B$  on the boundary of  $\mathcal{Q}$  such that segment  $AB$  has no common points with  $\mathcal{R}$  and has length greater than 521.