

30-th German Federal Mathematical Competition
1999/2000

Second Round

1. We are given $n \geq 3$ weights of masses $1, 2, 3, \dots, n$ grammes. Find all n for which it is possible to divide these weights into three groups with the same mass.
2. Prove that for every integer $n \geq 2$ there exist n different positive integers such that for any two of these integers a and b their sum $a + b$ is divisible by their difference $a - b$.
3. For each vertex of a given tetrahedron, a sphere passing through that vertex and the midpoints of the edges outgoing from this vertex is constructed. Prove that these four spheres pass through a single point.
4. Consider the sums of the form $\sum_{k=1}^{\infty} \varepsilon_k k^3$, where $\varepsilon_k \in \{-1, 1\}$. Is any of these sums equal to 0 if
 - (a) $n=2000$;
 - (b) $n=2001$?