

27-th German Federal Mathematical Competition 1996/97

First Round

1. Given 100 integers, is it always possible to choose 15 of them such that the difference of any two of the chosen numbers is divisible by 7? What is the answer if 15 is replaced by 16?
2. Determine all prime numbers p for which the system of equations

$$\begin{aligned}p + 1 &= 2x^2 \\ p^2 + 1 &= 2y^2\end{aligned}$$

has a solution in integers x, y .

3. A square S_a is inscribed in an acute-angled triangle ABC with two vertices on side BC and one on each of sides AC and AB . Squares S_b and S_c are analogously inscribed in the triangle. For which triangles are the squares S_a, S_b , and S_c congruent?
4. There are 10000 trees in a park, arranged in a square grid with 100 rows and 100 columns. Find the largest number of trees that can be cut down, so that sitting on any of the tree stumps one cannot see any other tree stump.