

25-th German Federal Mathematical Competition 1994/95

First Round

1. A game is played with two heaps of p and q stones. Two players alternate playing, with A starting. A player in turn takes away one heap and divides the other heap into two smaller ones. A player who cannot perform a legal move loses the game. For which values of p and q can A force a victory?
2. A line g and a point A outside g are given in a plane. A point P moves along g . Find the locus of the third vertices of equilateral triangles whose two vertices are A and P .
3. A natural number n is called *breakable* if there exist positive integers a, b, x, y such that $a + b = n$ and $\frac{x}{a} + \frac{y}{b} = 1$. Find all breakable numbers.
4. A number of unit discs are given inside a square of side 100 such that
 - (i) no two of the discs have a common interior point, and
 - (ii) every segment of length 10, lying entirely within the square, meets at least one disc.

Prove that there are at least 400 discs in the square.