10-th German Federal Mathematical Competition 1979/80

First Round

1. Six free cells are given in a row. Players A and B alternately write digits from 0 to 9 in empty cells, with A starting. When all the cells are filled, one considers the obtained six-digit number z. Player B wins if z is divisible by a prescribed natural number n, and loses otherwise. For which values of n not exceeding 20 can B win independently of his opponent's moves?

2. In a triangle ABC, the bisectors of angles A and B meet the opposite sides of the triangle at points D and E, respectively. Point P is arbitrarily chosen on the line DE. Prove that the distance of P from line AB equals the sum or the difference of the distances of P from lines AC and BC.

3. In the plane are given $2n+3$ points (where $n \in \mathbb{N}$), no three of which lie on a line and no four lie on a circle. Prove that there is a circle passing through three of the points and containing exactly n points in its interior.

4. Consider the sequence $a_1, a_2, a_3, \dots$ with $a_n = \frac{1}{n(n+1)}$. In how many ways can the number $\frac{1}{1980}$ be represented as the sum of finitely many consecutive terms of this seqeunce?

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