

7-th German Federal Mathematical Competition 1976/77

First Round

1. Among 2000 distinct positive integers there are equally many even and odd ones. The sum of the numbers is less than 3000000. Show that at least one of the numbers is divisible by 3.
2. A beetle crawls along the edges of an n -lateral pyramid, starting and ending at the midpoint A of a base edge and passing through each point at most once. How many ways are there for the beetle to do this? Show that the sum of the numbers of passed edges (over these ways) equals $1^2 + 2^2 + \dots + n^2$.
3. The number 50 is written as a sum of several positive integers (not necessarily distinct) whose product is divisible by 100. What is the largest possible value of this product?
4. In a cyclic quadrilateral, from the midpoint of each side a perpendicular to the opposite side is dropped. Prove that these perpendiculars have a common point.