6-th German Federal Mathematical Competition 1975/76

First Round

1. Nine lattice points (i.e. with integer coordinates) $P_1, P_2, \dots, P_9$ are given in space. Show that the midpoint of at least one of the segments P_iP_j, where $1 \le i < j \le 9$, is a lattice point as well.

2. Each of two opposite sides of a convex quadrilateral is divided into seven equal parts, and the corresponding division points are connected by a segment, thus dividing the quadrilateral into seven smaller quadrilaterals. Prove that the area of at least one of the small quadrilaterals equals $1/7$ of the area of the large quadrilateral.

3. a set S of rational numbers is ordered in a tree-diagram in such a way that each rational number $\frac{a}{b}$ (where a and b are coprime integers) has exactly two successors: $\frac{a}{a+b}$ and $\frac{b}{a+b}$. How should the initial element (the ultimate predecessor) be selected in order to arrange the set of all rationals r with $0 < r < 1$ in such an ordering? Give a procedure for determining the ordinal number of a rational number p/q in this ordering.

4. In a plane are given $n > 2$ distinct points. Some pairs of these points are connected by segments so that no two of the segments intersect. Prove that there are at most $3n - 6$ segments.