33-rd German Federal Mathematical Competition 2002/03

First Round

1. Given six consecutive positive integers, prove that there is a prime which divides exactly one of these integers.

2. Determine all triplets (x,y,z) of integers which satisfy the following equations:

$$x^3-4x^2-16x+60=y$$
$$y^3-4y^2-16y+60=z$$
$$z^3-4z^2-16z+60=x.$$

3. Points M and N are interior points of sides AB and BC respectively of a parallelogram $ABCD$ such that $AM=NC$. The segments AN and CM intersect at Q. Prove that DQ is the angle bisector of $\angle ADC$.

4. Find with proof all positive integers that are not representable in the form $\frac{a}{b}+\frac{a+1}{b+1}$, where a and b are positive integers.

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