30-th German Federal Mathematical Competition 1999/2000

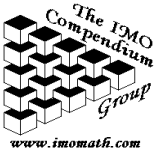
First Round

1. Two natural numbers have the same (decimal) digits in different order and have the sum $99\ldots99$. Is this possible if each of the numbers consists of (a) 1999 digits; (b) 2000 digits?

2. A 5-tuple $(1,1,1,1,2)$ has the property that the sum of any three of them is divisible by the sum of the remaining two. Is there a 5-tuple with this property whose all terms are distinct?

3. A convex quadrilateral $ABCD$ is inscribed in a semicircle with diameter AB. The diagonals AC and BD intersect at S, and the projection of S onto AB is denoted by T. Prove that ST bisects the angle $\angle CTD$.

4. A circular game board is divided into $n \geq 3$ sectors. Each sector is either empty or occupied by a marker. In each step one chooses an occupied sector, removes its marker and then switches each of the two adjacent sectors from occupied to empty or vice-versa. Starting with a single occupied sector, for which n is it possible to end up with all empty sectors after finitely many steps?



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