French IMO Selection Test 2006

First Day

1. A square $ABCD$ is inscribed in circle Γ. Let M be a point on the shorter arc CD. Line AM meets BD and CD respectively at P and R, and line BM meets AC and CD respectively at Q and S. Prove that PS and QR are perpendicular.

2. If positive numbers a,b,c satisfy $abc=1$, prove the inequality
$$\frac{a}{(a+1)(b+1)}+\frac{b}{(b+1)(c+1)}+\frac{c}{(c+1)(a+1)}\geq\frac{3}{4}$$
and find the cases of equality.

3. Suppose that a,b are positive integers such that b^n+n is a multiple of a^n+n for all $n\in\mathbb{N}$. Prove that $a=b$.

Second Day

4. Positive numbers are written in the cells of a $2\times n$ table so that in each of the n columns the sum of two numbers is 1. Prove that one can erase one number from each column in such a way that the sum of the remaining numbers in either row is at most $\frac{n+1}{4}$.

5. In a triangle ABC with $AC+BC=3AB$, the incircle with center I touches BC at D and CA at E. Let K and L be the points symmetric to D and E with respect to I. Prove that the points A,B,K,L lie on a circle.

6. The set $M=\{1,2,\dots,3n\}$ is partitioned into three subsets A,B,C of cardinality n. Show that there exist numbers a,b,c in three different subsets such that $a=b+c$.

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