47-th Czech and Slovak Mathematical Olympiad 1998

Third Round – Uherské Hradiště, March 22-25, 1998

Category A

1. Solve the equation $x \cdot [x \cdot [x \cdot [x]]] = 88$ in the set of real numbers. *(J. Šimša)*

2. Given any set of 14 (different) natural numbers, prove that for some k $(1 \le k \le 7)$ there exist two disjoint k-element subsets $\{a_1, \dots, a_k\}$ and $\{b_1, \dots, b_k\}$ such that
$$A = \frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_k} \quad \text{and} \quad B = \frac{1}{b_1} + \frac{1}{b_2} + \cdots + \frac{1}{b_k}$$
differ by less than 0.001, i.e. $|A - B| < 0.001$. *(J. Šimša)*

3. A sphere is inscribed in a tetrahedron $ABCD$. The tangent planes to the sphere parallel to the faces of the tetrahedron cut off four smaller tetrahedra. Prove that sum of all the 24 edges of the smaller tetrahedra equals twice the sum of edges of the tetrahedron $ABCD$. *(P. Leischner)*

4. For each date of year 1998, we calculate day$^{\text{month}}$ $-$ year and determine the greatest power of 3 that divides it. For example, for April 21 we get $21^4 - 1998 = 192483 = 3^3 \cdot 7129$, which is divisible by 3^3 and not by 3^4. Find all dates for which this power of 3 is the greatest. *(R. Kollár)*

5. A circle k and a point A outside it are given in the plane. Prove that all trapezoids, whose non-parallel sides meet at A, have the same intersection of diagonals. *(P. Leischner)*

6. Let a, b, c be positive numbers. Prove that a triangle with sides a, b, c exists if and only if the system of equations
$$\frac{y}{z} + \frac{z}{y} = \frac{a}{x}, \qquad \frac{z}{x} + \frac{x}{z} = \frac{b}{y}, \qquad \frac{x}{y} + \frac{y}{x} = \frac{c}{z}$$
has a real solution. *(P. Černek, J. Zhouf)*

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