# 41-st Czech and Slovak Mathematical Olympiad 1992

1. For a permutation $p(a_1,a_2,\dots,a_{17})$ of $1,2,\dots,17$, let $k_p$ denote the largest $k$ for which $a_1+\cdots+a_k < a_{k+1}+\cdots+a_{17}$. Find the maximum and minimum values of $k_p$ and find the sum $\sum_p k_p$ over all permutations $p$.

2. Let $S$ be the total area of a tetrahedron whose edges have lengths $a,b,c,d,e,f$. Prove that
$$S \le \frac{\sqrt{3}}{6}(a^2+b^2+\cdots+f^2).$$

3. Let $S(n)$ denote the sum of digits of $n \in \mathbb{N}$. Find all $n$ such that
$$S(n)=S(2n)=S(3n)=\cdots=S(n^2).$$

4. Solve the equation $\cos 12x = 5\sin 3x + 9\tan^2 x + \cot^2 x$.

5. The function $f:(0,1)\to\mathbb{R}$ is defined by
$$f(x)=\begin{cases} x & \text{if } x \text{ is irrational,}\\ \dfrac{p+1}{q} & \text{if } x=\dfrac{p}{q}, \text{ where } (p,q)=1.\end{cases}$$
Find the maximum value of $f$ on the interval $(7/8, 8/9)$.

6. Let $ABC$ be an acute triangle. The altitude from $B$ meets the circle with diameter $AC$ at points $P,Q$, and the altitude from $C$ meets the circle with diameter $AB$ at $M,N$. Prove that the points $M,N,P,Q$ lie on a circle.



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