6-th Czech–Polish–Slovak Match 2006

Žilina, Slovakia
June 26–27, 2006

1. Five points A,B,C,D,E lie in this order on a circle of radius r and satisfy $AC = BD = CE = r$. Prove that the orthocenters of triangles ACD, BCD, BCE form a rectangular triangle.

2. There are n children around a round table. Erika is the oldest among them and she has n candies, while no other child has any candy. Erika decided to distribute the candies according to the following rules. In every round, she chooses a child with at least two candies and the chosen child sends a candy to each of his/her two neighbors. (So in the first round Erika must choose herself). For which $n \geq 3$ is it possible to end the distribution after a finite number of rounds with every child having exactly one candy?

3. The sum of four real numbers is 9 and the sum of their squares is 21. Prove that these numbers can be denoted by a,b,c,d so that $ab - cd \geq 2$ holds.

4. Show that for every integer $k \geq 1$ there is a positive integer n such that the decimal representation of 2^n contains a block of exactly k zeros, i.e. $2^n = \dots a00\dots0b\dots$ with k zeros and $a,b \neq 0$.

5. Find the number of sequences $(a_n)_{n=1}^{\infty}$ of integers satisfying $a_n \neq -1$ and
$$a_{n+2} = \frac{a_n + 2006}{a_{n+1} + 1} \quad \text{for each } n \in \mathbb{N}.$$

6. Find out if there is a convex pentagon $A_1A_2A_3A_4A_5$ such that, for each $i = 1,\dots,5$, the lines A_iA_{i+3} and $A_{i+1}A_{i+2}$ intersect at a point B_i and the points B_1,B_2,B_3,B_4,B_5 are collinear. (Here $A_{i+5} = A_i$.)