Chinese IMO Team Selection Test 1995

Time: 4.5 hours each day.

First Day

1. Find the smallest prime number p that cannot be represented in the form $|3^a - 2^b|$ with a and b nonnegative integers.

2. Given are a fixed acute angle θ and a pair of circles that are internally tangent at A. A line l through A, not containing the centers of the circles, meets the larger circle again at B. For an arbitrary point M on the major arc AB, line AM intersects the smaller circle again at N, and P is the point on ray MB such that $\angle MPN = \theta$. Find the locus of P as M takes all possible positions.

3. Twenty-one students take a test with 15 true or false questions. It is known that for every two students there is a question which they both answered correctly. What is the minimum number of people that could have correctly answered the question which the most people were correct on?

Second Day

4. Let $S = \{(a_1, \dots, a_8) \mid \forall a_i = 0 \text{ or } 1\}$. For any elements $A = (a_1, \dots, a_8)$ and $B = (b_1, \dots, b_8)$ of S, we define the *distance* between A and B as $d(A,B) = \sum_{i=1}^{8} |a_i - b_i|$. At most, how many elements of S can be chosen so that the distance between any two of them is at least 5?

5. Two players play a game with a polynomial of degree at least 4:

$$x^{2n} + \textvisiblespace x^{2n-1} + \textvisiblespace x^{2n-2} + \cdots + \textvisiblespace x + 1 = 0,$$

   alternately filling in the blank coefficients with real numbers until all the blanks are filled up. The first player wins if the resulting polynomial has no real roots; otherwise, the second player wins. Which player has a winning strategy?

6. Given a quadratic polynomial $P(x)$, define the *weight* of a real interval $[a,b]$ as $P(b) - P(a)$. Prove that the interval $[0,1]$ can be split into black and white intervals such that the sum of weights of the black intervals is equal to the sum of weights of the white intervals. Is this result true if the degree of P is 3 or 5?

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