Chinese IMO Team Selection Test 1993

Time: 4.5 hours each day.

First Day – April 4

1. For an odd prime p, define $F(p) = \sum_{k=1}^{(p-1)/2} k^{120}$ and $f(p) = \frac{1}{2} - \left\{ \frac{F(p)}{p} \right\}$, where $\{x\}$ is the fractional part of x. Find the values which $f(p)$ can take.

2. Let $n \geq 2$ be an integer and let a,b,c,d be positive integers such that
$$a + c \leq n \quad \text{and} \quad \frac{a}{b} + \frac{c}{d} < 1.$$
Find the maximum value of $\frac{a}{b} + \frac{c}{d}$.

3. Prove that for any natural number n there exists a graph not containing triangles whose chromatic number is n.

Second Day – April 5

4. Find all integral solutions of the equation $2x^4 + 1 = y^2$.

5. Consider set $S = \{(x,y) \mid x = 1,2,\dots,1993,\ y = 1,2,3,4\}$ on the coordinate plane. Let T be a subset of S such that no four points in T form a square. Find the greatest possible number of elements of T.

6. The bisector of $\angle A$ meets the circumcircle of triangle ABC at D. Let I be the incenter, M be the midpoint of BC and P be the point symmetric to I with respect to M. Line DP meets the circumcircle of $\triangle ABC$ again at N. Prove that one of the segments AN, BN, CN equals the sum of the other two.

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