29-th Canadian Mathematical Olympiad 1997

1. How many pairs of positive integers (x,y) with $x \leq y$ are there such that $\gcd(x,y) = 5!$ and $\operatorname{lcm}(x,y) = 50!$?

2. Consider a finite set of closed intervals of length 1 whose union is the interval $[0,50]$. Prove that it is possible to select some of these intervals which are mutually disjoint and have the total length at least 25.

3. Prove that
$$\frac{1}{1999} < \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdots \frac{1997}{1998} < \frac{1}{44}.$$

4. Suppose that O is a point inside the parallelogram $ABCD$ such that $\angle AOB + \angle COD = 180^\circ$. Prove that $\angle OBC = \angle ODC$.

5. Write the sum
$$\sum_{k=0}^{n} \frac{(-1)^k \binom{n}{k}}{k^3 + 9k^2 + 26k + 24}$$
in the form $p(n)/q(n)$, where p and q are polynomials with integer coefficients.

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