26-th Canadian Mathematical Olympiad 1994

1. Evaluate the sum
$$\sum_{n=1}^{1994}(-1)^n\frac{n^2+n+1}{n!}.$$

2. Show that for every positive integer n the number $(\sqrt{2}-1)^n$ is of the form $\sqrt{m}-\sqrt{m-1}$ for some $m\in\mathbb{N}$.

3. Twenty-five men sit around a circular table. Every hour there is a vote, and each must respond *yes* or *no*. Each man behaves as follows: if his response on the n-th vote is the same as the response of at least one of the two people he sits between, then he will respond the same way on the $(n+1)-$th vote; otherwise he will change his response. Prove that how ever the men responded on the first vote, there will be a time after which nobody's response will ever change.

4. Let AB be a diameter of a circle Ω and P be any point not on the line AB. The lines PA and PB cut Ω again at U and V, respectively. (In the case of tangency U or V may coincide with A or B; also, if $P\in\Omega$ then $P=U=V$.) Suppose $s,t\geq 0$ are such that $PU=s\cdot PA$ and $PV=t\cdot PB$. Determine $\cos\angle APB$ in terms of s and t.

5. Let AD be the altitude of an acute-angled triangle ABC and let H be any interior point on AD. Lines BH and CH, intersect AC and AB at E and F, respectively. Prove that $\angle EDH=\angle FDH$.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com