25-th Canadian Mathematical Olympiad 1993

1. Find a triangle for which the three sides and an altitude are four consecutive integers and for which this altitude partitions the triangle into two right triangles with integer sides. Show that this triangle is unique.

2. Show that the number x is rational if and only if the sequence $x, x+1, x+2, \dots$ contains three terms that form a geometric progression.

3. In a triangle ABC, the medians from B and C are perpendicular. Prove that $\cot B + \cot C \geq \frac{2}{3}$.

4. A number of schools took part in a tenis tournament. No two players from the same school played against each other. A match between two boys or between two girls was called a *single* and that between a boy and a girl was called a *mixed single*. The numbers of boys and of girls differed by at most one. The numbers of singles and of mixed singles also differed by at most one. At most, how many schools were represented by an odd number of players?

5. The squence (y_n) is defined by $y_1 = 1$ and, for $k > 0$,

$$y_{2k} = \begin{cases} 2y_k & \text{if } k \text{ is even,} \\ 2y_k + 1 & \text{if } k \text{ is odd;} \end{cases} \qquad y_{2k+1} = \begin{cases} 2y_k & \text{if } k \text{ is odd,} \\ 2y_k + 1 & \text{if } k \text{ is even.} \end{cases}$$

   Show that every positive integer occurs in this sequence exactly once.

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