24-th Canadian Mathematical Olympiad 1992

1. Prove that $1 \cdot 2 \cdot \dots \cdot n$ is divisible by $1+2+\dots+n$ if and only if $n+1$ is not an odd prime.

2. For $x,y,z \geq 0$, prove the inequality
$$x(x-z)^2 + y(y-z)^2 \geq (x-z)(y-z)(x+y-z)$$
and find when equality holds.

3. Let U and V be interior points of the sides AB and CD respectively of a square $ABCD$. Lines AV and DU meet at P and lines BV and CU meet at Q. Determine all possible ways to select U and V so as to maximize the area of the quadrilateral $PUQV$.

4. Solve the equation $x^2 + \dfrac{x^2}{(x+1)^2} = 3$.

5. A deck of $2n+1$ consists of a joker and, for each $k = 1,2,\dots,n$, two cards marked with k. The cards are placed in a row, with the joker in the middle. For each $k = 1,\dots,n$, the two cards numbered k have exactly $k-1$ cards between them. Determine all $n \leq 10$ for which this arrangement is possible. For which values of n is it impossible?

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