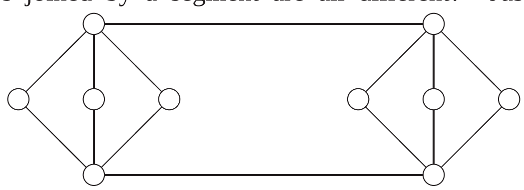
23-rd Canadian Mathematical Olympiad 1991

1. Show that the equation $x^2 + y^5 = z^3$ has infinitely many solutions in nonzero integers x, y, z.

2. For a positive integer n, find the sum of all positive integers whose base 2 representations have exactly n 1's and n 0's. (The first digit cannot be 0.)

3. Let $\mathscr{C}$ be a circle and P be a given point in the plane. Consider all possible chords of $\mathscr{C}$ determined by a line through P. Prove that the midpoints of these chords lie on a circle.

4. Is it possible to write some ten numbers from the set $\{0, 1, 2, \ldots, 14\}$ in the circles in the diagram in such a way that the absolute differences of two numbers joined by a segment are all different? Justify your answer.



5. An equilateral triangle of side length n is divided into n^2 unit equilateral triangles. Find the number $f(n)$ of parallelograms bounded by sides in the so obtained grid.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com