17-th Canadian Mathematical Olympiad 1985

May 1, 1985

1. The sides of a triangle have lengths $6,8$ and 10. Prove that there is exactly one line which simultaneously bisects the area and the perimeter of the triangle.

2. Prove or disprove that there exists an integer which is doubled when the initial digit is transfered to the end.

3. Let $\mathscr{P}_1$ and $\mathscr{P}_2$ be regular 1985-gons circumscribed about and inscribed in a given circle of perimeter c. The perimeters of $\mathscr{P}_1$ and $\mathscr{P}_2$ are x and y respetively. Prove that $x+y \geq 2c$. (You may assume that $\tan\theta \geq \theta$ for $0 \leq \theta < \pi/2$.)

4. Prove that $n!$ is divisible by 2^{n-1} if and only if $n = 2^{k-1}$ for some positive integers k.

5. Let $1 < x_1 < 2$. For $n = 1,2,\ldots$, define $x_{n+1} = 1 + x_n - x_n^2/2$. Prove that

$$|x_n - \sqrt{2}| < \frac{1}{2^n} \qquad \text{for all } n \geq 3.$$

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