11-th Canadian Mathematical Olympiad 1979

1. Let a, A_1, A_2, b be an arithmetic progression and a, G_1, G_2, b be a geometric progression, where $a, b > 0$. Show that $A_1A_2 \geq G_1G_2$.

2. It is known that the sum of angles of a triangle is constant. Prove, however, that the sum of dihedral angles of a tetrahedron is not constant.

3. Let $1 \leq a < b < c < d < e$ be integers. If $[m,n]$ denotes the lcm of m, n, prove that
$$\frac{1}{[a,b]} + \frac{1}{[b,c]} + \frac{1}{[c,d]} + \frac{1}{[d,e]} \leq \frac{15}{16}.$$

4. A dog standing at the center of a circular arena sees a rabbit at the wall. The rabbit runs around the wall and the dog pursues it along a unique path which is determined by running at the same speed and staying on the radial line joining the center of the arena and the rabbit. Show that the dog overtakes the rabbit just as it reaches a poiont one-quarter of the way around the arena.

5. A walk consists of a sequence of steps of length 1 in directions north, south, east or west. A walk is *self-avoiding* if it never passes through the same point twice. Let $f(n)$ denote the number of n-step self-avoiding walks which begin at the origin. Compute $f(n)$ for $n = 1, 2, 3, 4$ and show that for all n,
$$2^n < f(n) \leq 4 \cdot 3^{n-1}.$$

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