9-th Canadian Mathematical Olympiad 1977

1. If $f(x) = x^2 + x$, prove that the equation $4f(a) = f(b)$ has no solutions in positive integers a, b.

2. Let A be a point inside a given circle with center O ($A \neq O$). Find all points P on the circle for which $\angle OPA$ is a maximum.

3. Let N be an integer whose base b representation is 777. Find the smallest natural number b for which N is the fourth power of an integer.

4. Let $p(x) = a_n x^n + \cdots + a_1 x + a_0$ and $q(x) = b_m x^m + \cdots + b_1 x + b_0$ be two polynomials with integer coefficients. Suppose thaat all the coefficients of $p(x)q(x)$ are even, but not all of them are divisible by 4. Show that one of $p(x)$ and $q(x)$ has all even coeficients and the other has at least one odd coefficient.

5. A right circular cone of base radius 1 and slant height 3 is given. For a point P on the circumference of the base, the shortest path from P around the cone and back to P is drawn. Find the minimum distance from the vertex V of the cone to this path.

6. For $0 < u < 1$, the sequence (u_n) is defined by $u_1 = 1 + u$ and $u_{n+1} = \dfrac{1}{u_n} + u$ for $n \geq 1$. Show that $u_n > 1$ for all n.

7. A rectangular city is exactly m blocks long and n blocks wide (an $m \times n$ grid). A women lives in the southeast A woman lives in the southwest corner and works in the northeast. She walks to work each day, but on any given trip she makes sure that her path does not include any intersection twice. Show that the number $f(m,n)$ of different path she can take does not exceed 2^{mn}.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com