8-th Canadian Mathematical Olympiad 1976

1. Given four weights in geometric progression and an equal arm balance, show how to find the heaviest weight using the balance only twice.

2. A sequence (a_n) satisfies $a_0 = 1$, $a_1 = 2$ and

$$n(n+1)a_{n+1} = n(n-1)a_n - (n-2)a_{n-1}, \qquad n \geq 1.$$

Compute $\frac{a_0}{a_1} + \frac{a_1}{a_2} + \cdots + \frac{a_{50}}{a_{51}}$.

3. Two grade seven students were allowed to enter a chess tournament otherwise composed of grade eight students. Each contestant played once with each other contestant and received 1 point for a win, 0.5 for a tie and 0 for a loss. The two grade seven students together gained a total of eight points and each grade eight student scored the same number of points as his classmates. How many students from grade eight participated in the tournament? Is the solution unique?

4. Let C be a point on the diameter AB of a circle, and Q be a variable point on the circle. Let P be the point on the ray QC such that $AC/CB = QC/CP$. Describe the locus of P.

5. Prove that a positive integer is a sum of at least two ocnsecutive positive integers if and only if it is not a power of 2.

6. If A,B,C,D are four points in space such that $\angle ABC = \angle BCD = \angle CDA = \angle DAB = \pi/2$, prove that these points lie in a plane.

7. Let $P(x,y)$ be a polynomial in two variables such that $P(x,y) = P(y,x)$ for all x,y. Given that $(x-y)$ divides $P(x,y)$, prove that $(x-y)^2$ also divides $P(x,y)$.

8. Each of the 36 line segments joining 9 distinct points on a circle is colored either red or blue. Suppose that each triangle determined by three of the 9 points contains a red side. Prove that there are four points such that the six segments connecting them are all red.

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