

35-th Canadian Mathematical Olympiad 2003

March 26, 2003

1. Consider a standard twelve-hour clock whose hour and minute hands move continuously. Let m be an integer, with $1 \leq m \leq 720$. At precisely m minutes after 12:00, the angle made by the hour hand and minute hand is exactly 1° . Determine all possible values of m .
2. Find the last three digits of the number $2003^{2002^{2001}}$.
3. Find all real positive solutions to

$$\begin{aligned}x^3 + y^3 + z^3 &= x + y + z, \\x^2 + y^2 + z^2 &= xyz.\end{aligned}$$

4. Let k_1, k_2, k_3 be three circles with the common chord AB . A line l through A different from AB intersects the three circles at X, Y, Z , respectively. Show that the ratio $XY : YZ$ does not depend on l .
5. Let S be a set of n points in the plane such that any two points of S are at least 1 unit apart. Prove there is a subset T of S with at least $n/7$ points such that any two points of T are at least $\sqrt{3}$ units apart.