Bulgarian Mathematical Olympiad 1975, IV Round

First Day

1. Find all pairs of natural numbers (m,n) bigger than 1 for which 2^m+3^n is a square of whole number. (I. Tonov)

2. Let F is polygon the boundary of which is a broken line with vertices in the knots (units) of a given in advance regular square network. If k is the count of knots of the network situated over the boundary of F, and ℓ is the count of the knots of the network lying inside F, prove that if the surface of every square from the network is 1, then the surface S of F is calculated with the formulae:

$$S=\frac{k}{2}+l-1$$

(V. Chukanov)

3. Let $f(x)=a_0x^3+a_1x^2+a_2x+a_3$ is a polynomial with real coefficients ($a_0\neq 0$) and such that $|f(x)|\leq 1$ for every $x\in[-1,1]$. Prove that

   (a) there exist a costant c (one and the same for all polynomials with the given property), for which $|a_i|\leq c$, $i=0,1,\dots$.

   (b) $|a_0|\leq 4$.

(V. Petkov)

Second day

4. In the plane are given a circle k with radii R and the points $A_1,A_2,dots,A_n$, lying on k or outside k. Prove that there exist infinitely many points X from the given circunference for which

$$\sum_{i=1}^{n}A_iX^2\geq 2nR^2$$

Is there exist a pair of points on different sides of some diameter, X and Y from k, such that

$$\sum_{i=1}^{n}A_iX^2\geq 2nR^2 \text{ and } \sum_{i=1}^{n}A_iY^2\geq 2nR^2?$$

(H. Lesov)

5. Let *subbishop* (bishop is the figure moving only by a diagonal) is a figure moving only by diagonal but only in the next cells (suqres) of the chessboard. Find the maximal count of subbishops over a chessboard $n\times n$, no two of which are not attacking. (V. Chukanov)

6. Some of the walls of a convex polyhedron M are painted in blue, others are painted in white and there are no two walls with common edge. Prove that if the sum of surfaces of the blue walls is bigger than half surface of M then it may be inscribed a sphere in the polyhedron given (M). (H. Lesov)

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