Bulgarian Mathematical Olympiad 1974, IV Round

First Day

1. Find all natural numbers n with the following property: there exists a permutation $(i_1, i_2, \dots, i_n)$ of the numbers $1, 2, \dots, n$ such that, if on the circular table sit n people and for all $k = 1, 2, \dots, n$ the k-th person is moving in places in right, all people will sit on different places. (V. Drenski)

2. Let $f(x)$ anf $g(x)$ are non costant polynomials with integer positive coefficients, m and n are given natural numbers. Prove that there exists infinitely many natural numbers n for which the numbers the numbers

$$f(m^n) + g(0), f(m^n) + g(1), \dots, f(m^n) + g(k)$$

are composite. (I. Tonov)

3. (a) Find all real numbers p for which the inequality

$$x_1^2 + x_2^2 + x_3^2 \geq p(x_1x_2 + x_2x_3)$$

is true for all real numbers x_1, x_2, x_3.

(b) Find all real numbers q for which the inequality

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 \geq q(x_1x_2 + x_2x_3 + x_3x_4)$$

is true for all real numbers x_1, x_2, x_3, x_4.

(I. Tonov)

Second day

4. Find the maximal count of shapes that can be placed over a chessboard with size 8×8 in such a way that no three shapes are not on two squares, lying next to each other by diagonal parallel $A1 - H8$ ($A1$ is the lowest-bottom left corner of the chessboard, $H8$ is the highest-upper right corner of the chessboard). (V. Chukanov)

5. Find all point M lying into given acute-angled triangle ABC and such that the surface of the triangle with vertices on the foots of the perpendiculars drawn from M to the lines BC, CA, AB is maximal. (H. Lesov)

6. In triangle pyramid $MABC$ at least two of the plane angles next to the edge M are not equal to each other. Prove that if the angles bisectors of these angles form the same angle with the angle bisector of the third plane angle, the following inequality is true

$$8a_1b_1c_1 \leq a^2a_1 + b^2b_1 + c^2c_1$$

where a, b, c are sides of triangle ABC and a_1, b_1, c_1 are edges crossed respectively with a, b, c. (V. Petkov)

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