Bulgarian Mathematical Olympiad 1962, IV Round

1. It is given the expression $y = \frac{x^2-2x+1}{x^2-2x+2}$, where x is a variable. Prove that:

   (a) if x_1 and x_2 are two random values of x, and y_1 and y_2 are the respective values of y if $\le x_1 < x_2$, $y_1 < y_2$;

   (b) when x is varying y attains all possible values for which: $0 \le y < 1$

   (5 points)

2. It is given a circle with center O and radii r. AB and MN are two random diameters. The lines MB and NB intersects tangent to the circle at the point A respectively at the points M' and N'. M'' and N'' are the middlepoints of the segments AM' and AN'. Prove that:

   (a) around the quadrilateral $MNN'M'$ may be circumscribed a circle;

   (b) the heights of the triangle $M''N''B$ intersects in the middlepoint of the radii OA.

   (5 points)

3. It is given a cube with sidelength a. Find the surface of the intersection of the cube with a plane, perpendicular to one of its diagonals and which distance from the centre of the cube is equal to h. (4 points)

4. There are given a triangle and some its internal point P. x, y, z are distances from P to the vertices A, B and C. p, q, r are distances from P to the sides BC, CA, AB respectively. Prove that:

$$xyz = (q+r)(r+p)(p+q)$$

   (6 points)

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
LaTeX by Borislav Mirchev
www.imomath.com