28-th Bulgarian Mathematical Olympiad 1979

Third Round

First Day

1. Solve in positive integers the equation $\dfrac{19}{x^2}+\dfrac{79}{y^2}=\dfrac{z}{1979}$.

2. The incircle of triangle ABC is tangent to side BC at T. Prove that the incenter lies on the line that passes through the midpoints of AT and BC.

3. Suppose that $n \geq 5$ distinct triangles are such that any two have a side in common. Show that all n triangles have a common side.

Second Day

4. Let m and n be natural numbers such that $\sqrt{7}-\frac{m}{n}>0$. Prove that $\sqrt{7}-\frac{m}{n}>\frac{1}{mn}$.

5. Solve in real numbers the system

$$\begin{cases} \sqrt{1+(x+y)^2} = -y^6+2x^2y^3+4x^4 \\ \sqrt{2x^2y^2-x^4y^4} \geq 4x^2y^3+5x^3. \end{cases}$$

6. In a regular n-gonal pyramid, α is the dihedral angle between a lateral face and the base, and β is the dihedral angle between two adjacent lateral faces.

   (a) Prove that $\cos\frac{\beta}{2}=\sin\frac{\pi}{n}\sin\alpha$.

   (b) Evaluate $\lim\limits_{\alpha\to 0}\dfrac{\pi-\beta}{\alpha}$.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com