Bulgarian Mathematical Olympiad 1972, III Round

First Day

1. Prove that for every integer number and natural number n the number $a^N - a$ is divisible by 13 where $N = 2^{2^n} - 3$. (6 points, Hr. Le Lesov)

2. Prove inequality:

$$1 + \frac{1}{1!\sqrt{2!}} + \frac{1}{2\sqrt{2!}\sqrt[3]{3!}} + \cdots + \frac{1}{(n-1)\sqrt[n-1]{(n-1)!}\sqrt[n]{n}} > \frac{2(n^2+n-1)}{n(n+1)}$$

where n is a natural number, greater than 1. (7 points, Hr. Lesov)

3. Find all positive integer values of n for which whole plane may be covered with network that consists of regular n-gons. (7 points, Hr. Lesov)

Second day

4. The opposite sides AB and CD of inscribed in the circle k quadrilateral $ABCD$ intersects at a point M. Tangent MN (N belongs to k) is parallel to the diagonal AC. NB intersects AC at the point P. Prove that the lines AN, DB and PM intersects at a common point.

(6 points, Hr. Lesov)

5. On the sides BC, CA, AB of acute angled triangle ABC externally are constructed squares which centers are denoted by M, N, P. Prove the inequality:

$$[MNP] \geq \left(1 + \frac{\sqrt{3}}{2}\right)[ABC]$$

(6 points, Hr. Lesov)

6. It is given a pyramid with base n-gon, circumscribed around a circle with center O, which is orthogonal projection of the vertex of the pyramid to the plane of the base of the pyramid. Prove that the orthogonal projections of O to the walls of the pyramid lies on the common circle.

(8 points, Hr. Lesov)

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