Bulgarian Mathematical Olympiad 1968, III Round

First Day

1. Find four digit number $\overline{1xyz}$, if two of the numbers $\overline{xz}$, $\overline{yx}+1$, $\overline{zy}-2$ are divisible by 7 and $x+2y+z=29$. (6 Points)

2. Find the numbers A, B, C in such a way that for every natural number n is true the following equality

$$\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\cdots+\frac{n}{2^n}=\frac{An+B}{2^n}+C$$

(7 Points)

3. Solve the inequality

$$(1-\cos x)(1+\cos 2x)(1-\cos 3x)<\frac{1}{2}$$

(7 Points)

Second day

4. The points A, B, C and D are sequential vertices of regular polygon and the following equality is satisfied

$$\frac{1}{AB}=\frac{1}{AC}+\frac{1}{AD}$$

How much sides the polygon have ? (6 Points)

5. In a triangle ABC over the median CM is chosen a random point O. The lines AO and BO intersects the sides BC and AC at the points K and L respectively. Prove that if $AC>BC$ then $AK>BL$. (6 Points)

6. The base of pyramid $SABCD$ (with base $ABCD$) is a quadrilateral with mutually perpendicular diagonals. The orthogonal projection of the vertex S over the base of the pyramid coincides with the intersection point of the diagonals AC and BD. Prove that the orthogonal projections of the point O over the walls of the pyramid lies over a common circle. (8 Points)

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