Bulgarian Mathematical Olympiad 1967, III Round

First Day

1. Find four digit number which on division by 139 gives a remainder 21 and on division by 140 gives a remainder 7. (7 Points)

2. There are given 12 numbers $a_1, a_2, \dots, a_{12}$ satisfying the conditions:

$$a_2(a_1 - a_2 + a_3) < 0$$
$$a_3(a_2 - a_3 + a_4) < 0$$
$$\dots$$
$$a_{11}(a_{10} - a_{11} + a_{12}) < 0$$

   Prove that among these numbers there are at least three positive and three negative. (6 points)

3. On time of suspension of arms around round (circular) table are situated few knights from two enemy's camps. It is known that the count of knights with an enemy on its right side is equal to the count of knights with a friend on its right side. Prove that the total count of the knights situated around the circular table is divisible by 4. (7 points)

Second day

4. In the triangle ABC from the foot of the altitude CD is drawn a perpendicular DE to the side BC. On the line DC is taken point H for which $DH : HE = DB : DA$. Prove that the segments CH and AE are mutually perpendicular. (6 Points)

5. Prove that for each acute angled triangle is true the following inequality:

$$m_a + m_b + m_c \leq 4R + r$$

   (8 Points)

6. From the tetrahedrons $ABCD$ with a given volume V for which:

$$AC \perp CD \perp DB \perp AC$$

   find this one with the smallest radii of the circumscribed sphere. (6 Points)

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